The Duke TIP *Algebra 1* course corresponds to a high school Algebra 1 course and is designed for gifted students in grades seven through nine who want to build their algebra skills before taking algebra in high school or who want to take a self-paced algebra 1 course. This gifted curriculum provides a range of development opportunities from novice gifted (slightly above grade level) to advanced gifted (more than two grades above grade level).

The curriculum in this course pulls from standard objectives common to many states and from the Common Core Mathematics Standards.

## Lesson 1

After completing **Lesson 1: Foundations of Algebra**, the student will be able to do the following:

- Define *set*, *subset*, *intersection*, *union*, and *the empty set*.
- Write a set using set notation.
- Indicate whether a given value is an element of a set.
- Indicate whether a given set is a subset of another set.
- Find the intersection and union of two given sets.
- Define *natural numbers*, *whole numbers*, *integers*, *rational numbers*, *irrational numbers*, and *real numbers*. (N.RN.3)
- Use symbols to compare numbers, including equal, not equal, less than, greater than, less than or equal to, and greater than or equal to.
- Define *absolute value*.
- Add, subtract, multiply, and divide integers.
- Write a number in exponential notation.
- Find the base and the exponent of a number written in exponential notation.
- Multiply exponential expressions using the multiplication property of exponents.
- Simplify exponential expressions using the power of a power property of exponents.
- Simplify exponential expressions using the power of a product property of exponents.
- Find the square root of a given number.
- State and apply the order of operations.
- State and use the identity property of addition.
- State and use the identity property of multiplication.
- State and use the inverse property of addition.
- State and use the inverse property of multiplication.
- State and use the commutative property of addition.

- State and use the commutative property of multiplication.
- State and use the associative property of addition.
- State and use the associative property of multiplication.
- State and use the distributive property.

## Lesson 2

After completing **Lesson 2: Solving Linear Equations**, the student will be able to do the following:

- Define *algebraic expression*, *term*, *variable*, *constant*, *coefficient*, and like terms.
- Evaluate algebraic expressions for given values of variables.
- Simplify algebraic expressions by combining like terms.
- Simplify algebraic expressions using the distributive property.
- Define *equation*.
- Define *solution*.
- Use the addition property of Equality and the multiplication property of Equality to solve linear equations. (A.REI.1, A.REI.3)
- Determine if a given value is a solution to a given equation.
- Define *linear equation in one variable*.
- Define *identity*.
- Determine if an equation has one solution, no solution, or multiple solutions. (A.CED.3)
- Define *formula*.
- Solve a given formula for one of the variables in terms of the other variables. (A.CED.4, A.REI.3)
- Translate English expressions into algebraic expressions.
- Use algebra to solve linear word problems, including percent, perimeter, volume, and constant velocity. (N.Q.1)

## Lesson 3

After completing **Lesson 3: Introduction to Graphing**, the student will be able to do the following:

- Define origin, x-*axis*, y-*axis*, and *quadrants*.
- Graph ordered pairs on the *xy*-plane. (A.REI.10)
- Give the coordinates of a point graphed on the *xy*-plane.
- Find average rate of change for linear functions. (F.IF.6)

- Given two points on a line, determine the slope of the line. (F.LE.1a)
- Distinguish between lines that have positive slope, negative slope, zero slope, and undefined slope. (F.IF.4)
- Define *linear equation in two variables*.
- Determine if a given ordered pair is a solution to a given linear equation.
- Find the *x*-intercept and the *y*-intercept for a given linear equation. (F.IF.4)
- Graph a linear equation using the slope and the *y*-intercept. (F.IF.4)
- Graph a linear equation using the *x*- and *y*-intercepts. (F.IF.4, F.IF.7a)
- Graph horizontal and vertical lines given the equation of the line.
- Write a linear equation in slope-intercept form.
- Find the slope and *y*-intercept of a given linear equation. (F.IF.4)
- Write the equation of a line given two points on the line. (F.LE.2)
- Find the equation of a line given the slope and a point on the line. (F.LE.2)
- Find the equation of a horizontal line passing through a given point.
- Find the equation of a vertical line passing through a given point.
- Determine the slope of parallel lines, and perpendicular lines.
- Solve word problems involving consecutive numbers and averages. (A.CED.1, A.CED.2, F.BF.1b, F.LE.5)

## Lesson 4

After completing **Lesson 4: Inequalities and Systems of Equations**, the student will be able to do the following:

- Use the addition property of inequality and the multiplication property of inequality to solve linear inequalities.
- Graph solutions to linear inequalities on a number line.
- Solve word problems involving inequalities.
- Solve compound inequalities.
- Graph solutions to compound inequalities on a number line.
- Graph a linear inequality in two variables.
- Find the solution of a system of equations by graphing. (A.REI.6)
- Determine if a system has one unique solution, no solutions, or an infinite number of solutions.
- Find the solution of a system of equations by substitution. (A.REI.6)
- Find the solution of a system of equations by elimination. (A.REI.5, A.REI.6)

- Solve systems of equations problems involving numbers, interest, mixture, and money. (A.CED.3)
- Solve systems of inequalities by graphing. (A.REI.12)
- Determine if a system of inequalities has no solution. (A.CED.3)
- Solve word problems involving velocity, mixtures, and investments. (A.CED.1, A.CED.2)

## Lesson 5

After completing **Lesson 5: Functions and Exponents**, the student will be able to do the following:

- Determine if a given relation is a function.
- Use the vertical line test to determine if a given graph is a function.
- Write functions using function notation. (F.IF.2)
- Evaluate functions for given values. (F.IF.2)
- Define *function*, *domain*, and *range*. (F.IF.1)
- Determine the domain and range of a given function. (F.IF.5)
- Use interval notation and inequalities to represent domain and range.
- Multiply exponential expressions using the multiplication property of exponents.
- Divide exponential expressions using the division property of exponents.
- Simplify exponential expressions using the power of a power property of exponents.
- Simplify exponential expressions using the power of a product property of exponents.
- Simplify exponential expressions using the power of a quotient property of exponents.
- Simplify exponential expressions using the zero power property of exponents.
- Graph exponential functions. (F.IF.7e, F.IF.9, F.LE.1a, F.LE.2)
- Solve word problems involving exponential growth and decay. (F.BF.1a, F.LE.1c, F.LE.5, F.IF.8b)

## Lesson 6

After completing **Lesson 6: Polynomials and Factoring**, the student will be able to do the following:

- Define *term*, *polynomial*, *coefficient*, *monomial*, *binomial*, and *trinomial*. (A.SSE.1a)
- Find the degree of each term of a given polynomial.
- Find the degree of a given polynomial.
- Write the terms of a polynomial in descending order.

- Add and subtract polynomials by combining like terms. (A.APR.1)
- Multiply monomials and polynomials by monomials. (A.APR.1)
- Divide monomials and polynomials by monomials. (A.APR.1)
- Multiply binomials using the FOIL method. (A.APR.1)
- Square binomials. (A.APR.1)
- Multiply the sum and difference of two terms to get a difference of squares. (A.APR.1)
- Multiply polynomials by polynomials. (A.APR.1)
- Find the greatest common factor (GCF) of a group of monomials.
- Factor the GCF from a polynomial.
- Factor polynomials by grouping. (A.SSE.1b)
- Factor trinomials with leading coefficient 1.
- Factor trinomials with leading coefficient $\neq$ 1.
- Factor perfect square trinomials.
- Factor difference of two squares.
- Factor sum or difference of two cubes.
- Factor polynomials using various methods. (A.SSE.2)
- State and apply the zero product property.

## Lesson 7

After completing **Lesson 7: Rational Expressions**, the student will be able to do the following:

- Define *rational expression*.
- Evaluate rational expressions for given values.
- Reduce rational expressions to lowest terms.
- Multiply and divide rational expressions.
- Add and subtract rational expressions.
- Simplify complex fractions.
- Determine if a given value is a solution to a rational equation.
- Solve rational equations, including proportions.
- Determine if solutions are viable.
- Determine if a given value is a solution to a rational inequality.
- Find the domain of given rational functions.
- Solve a formula containing a rational expression for a given variable.
- Solve word problems involving work; distance, rate, and time; and resistors. (A.CED.1)

## Lesson 8

After completing **Lesson 8: Roots and Radicals**, the student will be able to:

- Define *radical* and *radicand*.
- Evaluate expressions of the form $a^{\frac{m}{n}}$.
- Define *the nth root of a number*.
- Use properties of exponents to simplify expressions with rational exponents. (N.RN.1, A.SSE.3c, F.IF.8b)
- Evaluate radical expressions with exponents equal to zero. (N.RN.1)
- Simplify radical expressions that contain variables and negative radicands.
- Use the multiplication property of radicals to simplify radicals. (N.RN.2)
- Simplify a square root by using prime factorization.
- Use the division property of radicals to simplify radicals. (N.RN.2)
- Define *like radical terms*.
- Add and subtract radicals.
- Multiply radical expressions that contain two or more terms.
- Define *conjugate*.
- Divide radical expressions by rationalizing the denominator.
- Solve radical equations.
- Use the Pythagorean theorem to solve equations. (A.CED.1)
- Solve word problem involving meteorology.

## Lesson 9

After completing **Lesson 9: Quadratic Equations**, the student will be able to do the following:

- Define *quadratic equation* and *standard form of a quadratic equation*.
- Determine if given values are solutions to a given quadratic equation.
- Write quadratic equations in standard form.
- Solve quadratic equations by factoring and using the zero product property. (A.SSE.3a, A.REI.4b, F.IF.8a)
- Solve quadratic equations by using the square root property. (A.REI.4b)
- Solve quadratic equations by completing the square. (A.SSE.3b, A.REI.4a, A.REI.4b, F.IF.8a)
- Solve quadratic word problems. (F.BF.1a)
- Solve quadratic equations using the quadratic formula. (A.REI.4a)

- Use the discriminant to determine the nature of the solutions of a quadratic equation. (A.REI.4b)
- Find the polynomial equation given the roots of the equation.
- Distinguish between quadratic equations which have one (double) solution, two solutions, or zero solutions in the real numbers.
- Solve word problem involving geometry, motion, and projectiles. (F.BF.1b)

## Lesson 10

After completing **Lesson 10: Nonlinear Functions**, the student will be able to do the following:

- Find the domain and range of various functions, including linear, quadratic, rational, and exponential. (F.IF.5)
- Use interval notation and inequalities to represent domain and range.
- Compare the graphs of linear functions to basic nonlinear functions, including quadratic, cubic, and absolute value functions. (F.LE.3, F.IF.7b)
- Graph piece-wise functions. (F.IF.7b)
- Find the $x$- and $y$-intercepts of a given quadratic function.
- Graph quadratic functions. (F.IF.4, F.IF.7a)
- Complete the square to find the vertex of a quadratic function. (A.SSE.3b)
- Use the formula for the vertex to write a quadratic function in standard form.
- Find the equation of a circle, given the center and the radius.
- Graph a circle given the equation.
- Use completing the square to find the center and radius of a given circle.
- Solve a system of equations consisting of a linear equation and a quadratic equation in two variables. (A.CED.2, A.REI.7)
- Use technology (graphs, tables, approximations) to solve systems of equations. (A.REI.11)

## Lesson 11

After completing **Lesson 11: Functions, Inverses, and Translations**, the student will be able to do the following:

- Add, subtract, multiply, and divide functions.
- Evaluate functions for a given value.
- Find the composition of two functions.
- Define *one-to-one functions*.

- Use the horizontal line test to determine if a given graph is one-to-one.
- Find the inverse of a function algebraically. (F.BF.4a)
- Find the inverse of a function by reflecting its graph about the line $y = x$. (F.BF.4a)
- Graph vertical and horizontal translations of standard graphs. (F.BF.3, F.IF.9)

## Lesson 12

After completing **Lesson 12: Modeling and Statistics**, the student will be able to do the following:

- Define *arithmetic sequence*. (F.IF.3, F.LE.1b)
- Write an arithmetic sequence as a linear function. (F.IF.3, F.BF.2, F.LE.2)
- Define *geometric sequence*. (F.IF.3, F.LE.1c)
- Write a geometric sequence as an exponential function. (F.BF.2, F.LE.2)
- Represent data using histogram or box plots. (S.ID. 1)
- Compare data sets using measures of central tendency. (S.ID.2, S.ID.5)
- Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers. (S.ID.3)
- Use scatterplots to represent data. (S.ID.6)
- Calculate the line of best fit for a given data set and calculate the correlation coefficient for that data set. (S.ID.6a, S.ID.6b, S.ID.6c, S.ID.7, S.ID.8, S.ID.9, F.BF.1b)
- Determine the best representation for given data sets. (N.Q.1, N.Q.2, N.Q.3, S.ID.9)