Problem: Using exactly four 4s, represent the numbers 1 to 25 using common mathematical symbols.

In this activity, you are to use the operations: addition, subtraction, multiplication, division, and square root, along with grouping symbols, to represent the numbers 1 to 25 using four 4s. You can also use square root and factorial*. Several examples for the number 1 are given below:

\[
1 = (4 \cdot 4) \div (4 \cdot 4) \quad 1 = 44 \div 44 \quad 1 = (4 + 4) \div (4 + 4) \quad 1 = 4 \cdot 4 \div 4 \div 4
\]

In the second example above, you’ll notice that concatenation (writing 44) is allowed in this activity.

\[
\begin{align*}
2 &= \quad 14 = \\
3 &= \quad 15 = \\
4 &= \quad 16 = \\
5 &= \quad 17 = \\
6 &= \quad 18 = \\
7 &= \quad 19 = \\
8 &= \quad 20 = \\
9 &= \quad 21 = \\
10 &= \quad 22 = \\
11 &= \quad 23 = \\
12 &= \quad 24 = \\
13 &= \quad 25 = \\
\end{align*}
\]

*If you are not familiar with factorial, it is represented by an exclamation point “!” and it tells us to multiply the number by itself and by every natural number less than it. For example, \(10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1\). For this activity, you can use \(4!\), which is \(4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24\).

1. If you enjoyed this activity, you don’t have to stop with 25. You can continue with other integers, and see how far you can get.

As the numbers get bigger, you may find that you need more operations. You may need to use exponents (for example, \(4^4 = 256\)) or decimals. Note: If you are writing 0.4, you’ll need to write it as .4 since 0 is a digit, and using it would violate the rules!

This problem has intrigued mathematicians for years. Currently, solutions have been found from 0 to 40,000. However, in order to find values that high, the rules have been changed to allow more mathematical operations. For other variations of operations that are acceptable, see http://www.dwheeler.com/fourfours.

2. You can also try the same activity using five fives.

3. And if that’s not enough, try doing it using six sixes.

4. Another similar activity is All the Kings’ Digits (Terry) found at http://mathforum.org/~terry/kings. This activity requires you to use all the digits \(\{0, 1, 2, 3, \ldots, 9\}\) to make the number 100. The rules are similar to those of Four 4s, but one major difference is that you must make sure that the 0 is essential. For example, if you use the digits 1, 2, 3, \ldots, 9 to equal 100 and then you add 0, then that 0 is non-essential. It should be that if you remove the 0, you will get an answer other than 100.